Mechanics Questions

Q1.

A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m.

The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i)   the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)

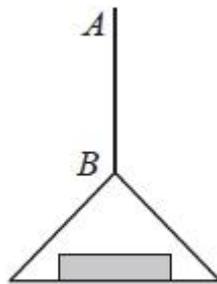**(Total for question = 7 marks)**

Q2.

Figure 1

A vertical rope AB has its end B attached to the top of a scale pan. The scale pan has mass 0.5 kg and carries a brick of mass 1.5 kg, as shown in Figure 1. The scale pan is raised vertically upwards with constant acceleration 0.5 m s^{-2} using the rope AB. The rope is modelled as a light inextensible string.

(a) Find the tension in the rope AB.

(3)

(b) Find the magnitude of the force exerted on the scale pan by the brick.

(3)

(Total for question = 6 marks)

Q3.

A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leq t \leq 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for question = 9 marks)

Q4.

A stone is projected vertically upwards from a point A with speed u m s^{-1}. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

(Total 11 marks)

Q5.

At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds, the velocity of P is v m s^{-1} in the positive x direction, where

$$v = 3t^2 - 16t + 21$$

The particle is instantaneously at rest when $t = t_1$ and when $t = t_2$ ($t_1 < t_2$).

(a) Find the value of t_1 and the value of t_2.

(2)

(b) Find the magnitude of the acceleration of P at the instant when $t = t_1$.

(3)

(c) Find the distance travelled by P in the interval $t_1 \leq t \leq t_2$.

(4)

(d) Show that P does not return to O.

(3)

(Total for question = 12 marks)

Statistics Questions

Q6.

A company director decides to survey staff about changes to the company calendar. The company has staff in 4 different job roles

72 managers, 108 drivers, 180 administrators and 360 warehouse staff.

The director decides to take a stratified sample.

(a) Write down one advantage of using a stratified sample rather than a simple random sample for this survey.

(1)

(b) Find the number of staff in each job role that will be included in a stratified sample of 40 staff.

(3)

(c) Describe how to choose managers for the stratified sample.

(2)

(Total for question = 6 marks)

Q7.

A factory produces components. Each component has a unique identity number and it is assumed that 2% of the components are faulty. On a particular day, a quality control manager wishes to take a random sample of 50 components.

(a) Identify a sampling frame.

(1)

The statistic F represents the number of faulty components in the random sample of size 50.

(b) Specify the sampling distribution of F.

(2)

(Total 3 marks)

Q8.

The following table summarises the times, t minutes to the nearest minute, recorded for a group of students to complete an exam.

Time (minutes) t	11 – 20	21 – 25	26 – 30	31 – 35	36 – 45	46 – 60
Number of students f	62	88	16	13	11	10

[You may use $\sum f t^2 = 134281.25$]

(a) Estimate the mean and standard deviation of these data.

(5)

(b) Use linear interpolation to estimate the value of the median.

(2)

(c) Show that the estimated value of the lower quartile is 18.6 to 3 significant figures.

(1)

(d) Estimate the interquartile range of this distribution.

(2)

(e) Give a reason why the mean and standard deviation are not the most appropriate summary statistics to use with these data.

(1)

The person timing the exam made an error and each student actually took 5 minutes less than the times recorded above. The table below summarises the actual times.

Time (minutes) t	6 – 15	16 – 20	21 – 25	26 – 30	31 – 40	41 – 55
Number of students f	62	88	16	13	11	10

(f) Without further calculations, explain the effect this would have on each of the estimates found in parts (a), (b), (c) and (d).

(3)

(Total 14 marks)

Q9.

The mark, x, scored by each student who sat a statistics examination is coded using

$$y = 1.4x - 20$$

The coded marks have mean 60.8 and standard deviation 6.60

Find the mean and the standard deviation of x.

(4)

(Total 4 marks)

Q10.

The histogram in Figure 1 shows the time, to the nearest minute, that a random sample of 100 motorists were delayed by roadworks on a stretch of motorway.
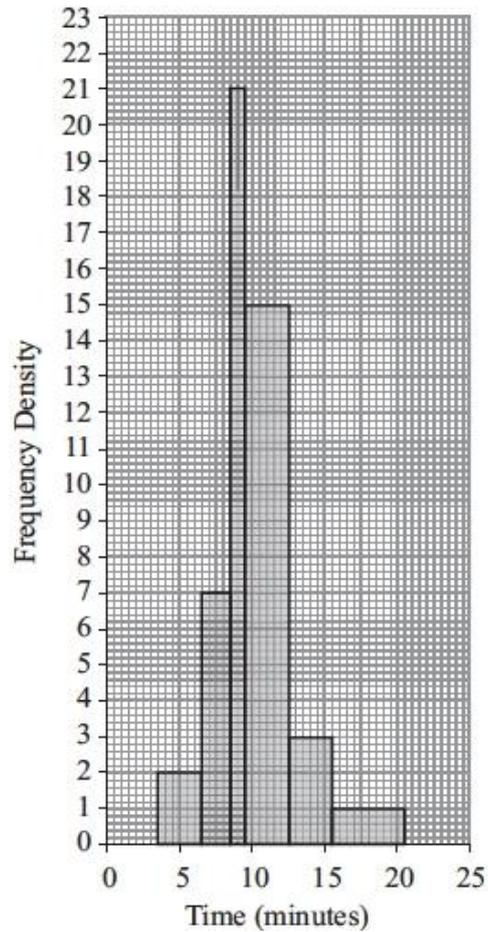

Figure 1

(a) Complete the table.

Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	
9	21
10 – 12	45
13 – 15	9
16 – 20	

(2)

(b) Estimate the number of motorists who were delayed between 8.5 and 13.5 minutes by the roadworks.
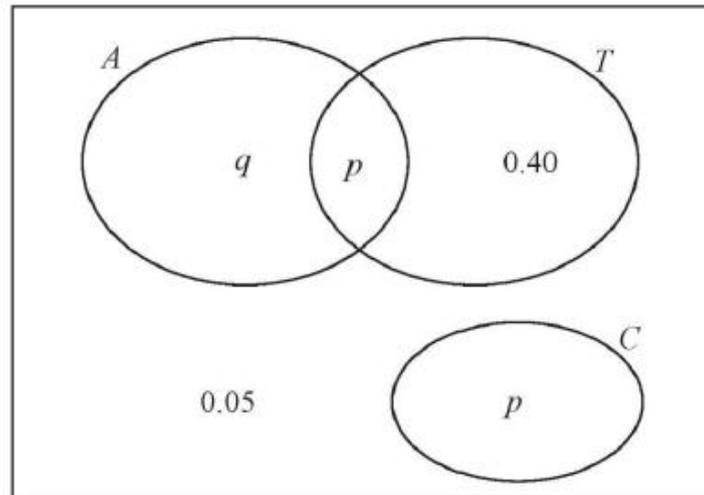
(2)

(Total 4 marks)

Q11.

The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.
T represents the event that a student takes part in Tennis.
C represents the event that a student takes part in Cricket.
p and q are probabilities.

The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.

(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

(Total for question = 5 marks)

Q12.

The proportion of houses in Radville which are unable to receive digital radio is 25%.
In a survey of a random sample of 30 houses taken from Radville, the number, X, of houses which are unable to receive digital radio is recorded.

(a) Find $P(5 \leqslant X < 11)$

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

(b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

(Total for question = 8 marks)

Q13.

A test statistic has a distribution B(25, p).

Given that

$$H_0 : p = 0.5 \quad H_1 : p \neq 0.5$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)

(Total 5 marks)

Q14.

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that
(i) exactly 6 ask for water with their meal,
(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geq 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

(Total 8 marks)

Q15.

(a) The discrete random variable $X \sim$ B(40, 0.27)

Find $P(X \geq 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(d)   Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e)   Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f)   Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

(Total for question = 9 marks)