Year 12 Pure 1 exam practice

1.

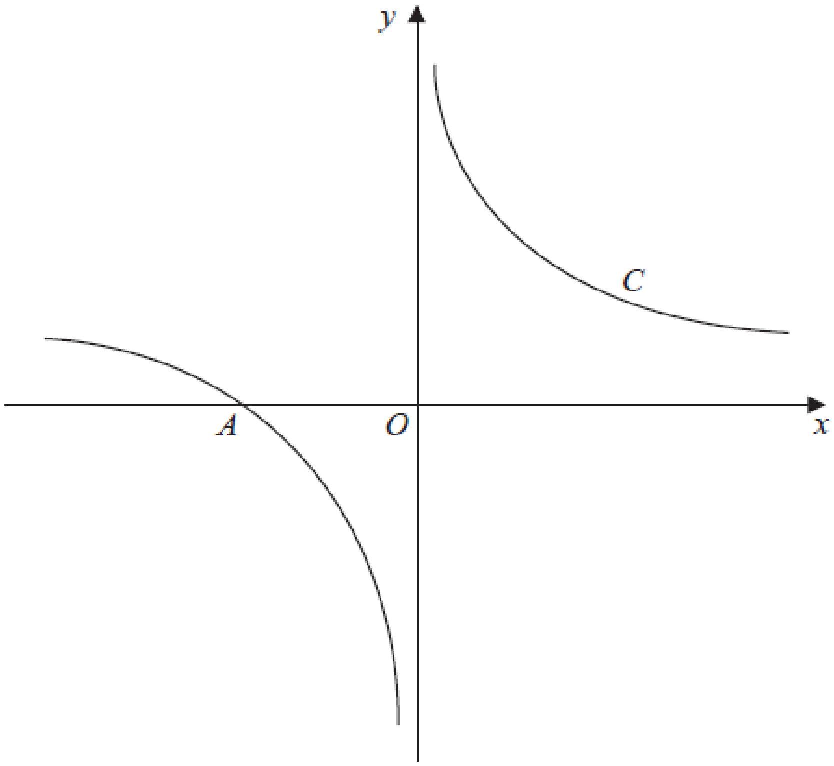

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \qquad x \neq 0.$$

The curve C crosses the x-axis at the point A.

(a) State the x-coordinate of the point A. **(1)**

The curve D has equation $y = x^2(x - 2)$, for all real values of x.

(b) On a copy of Figure 1, sketch a graph of curve D. Show the coordinates of each point where the curve D crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1$$

(1)(Total 5 marks)

2. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$ **(3)**

(b) Find the set of possible values of k. **(4) (Total 7 marks)**

3. $f(x) = 2x^3 - 7x^2 + 4x + 4.$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total 6 marks)

4. (a) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

(Total 8 marks)

5. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2 (11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

(Total 9 marks)

Turn over

6. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2,$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x + 5} - 7(2^x) = 0,$$

giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total 7 marks)

7. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(Total 4 marks)

8. A curve with equation $y = f(x)$ passes through the point (4, 9).

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0,$$

(a) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$.

(b) Find the x-coordinate of P.

(5)

(Total 10 marks)

9.

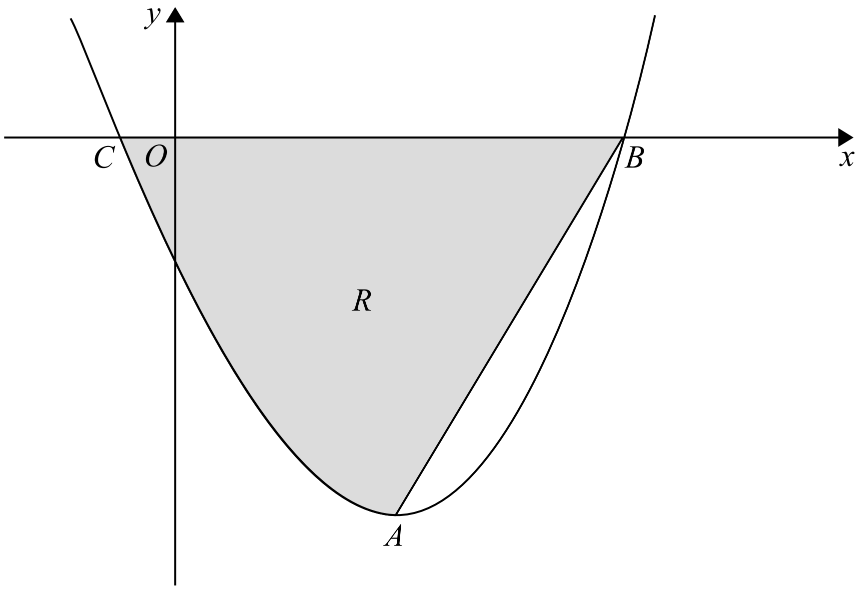

Figure 3 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \qquad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points B (2, 0) and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

(Total 10 marks)

The end….