

### Differentiation C3 Questions

1. The curve  $C$  has equation  $y = 4x^{\frac{3}{2}} - \ln(5x)$ , where  $x > 0$ . The tangent at the point on  $C$  where  $x = 1$  meets the  $x$ -axis at the point  $A$ .

Prove that the  $x$ -coordinate of  $A$  is  $\frac{1}{5} \ln(5e)$ .

2. A curve has equation  $7x^2 + 48xy - 7y^2 + 75 = 0$ .

$A$  and  $B$  are two distinct points on the curve. At each of these points the gradient of the curve is equal to  $\frac{2}{11}$ .

- (a) Use implicit differentiation to show that  $x + 2y = 0$  at the points  $A$  and  $B$ .
- (b) Find the coordinates of the points  $A$  and  $B$ .

3. The curve  $C$  has equation  $y = f(x)$ , where

$$f(x) = 3 \ln x + \frac{1}{x}, \quad x > 0.$$

The point  $P$  is a stationary point on  $C$ .

- (a) Calculate the  $x$ -coordinate of  $P$ .
- (b) Show that the  $y$ -coordinate of  $P$  may be expressed in the form  $k - k \ln k$ , where  $k$  is a constant to be found.

The point  $Q$  on  $C$  has  $x$ -coordinate 1.

- (c) Find an equation for the normal to  $C$  at  $Q$ .

The normal to  $C$  at  $Q$  meets  $C$  again at the point  $R$ .

- (d) Show that the  $x$ -coordinate of  $R$

(i) satisfies the equation  $6 \ln x + x + \frac{2}{x} - 3 = 0$ ,

- (ii) lies between 0.13 and 0.14.

4. (a) Differentiate with respect to  $x$

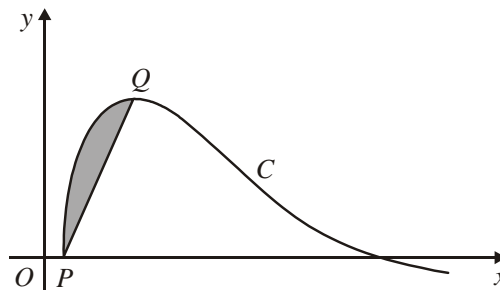
(i)  $3 \sin^2 x + \sec 2x,$

(ii)  $\{x + \ln(2x)\}^3.$

Given that  $y = \frac{5x^2 - 10x + 9}{(x-1)^2}, x \neq 1,$

(b) show that  $\frac{dy}{dx} = -\frac{8}{(x-1)^3}.$

5.



The figure above shows a sketch of part of the curve  $C$  with equation

$$y = \sin(\ln x), \quad x \geq 1.$$

The point  $Q$ , on  $C$ , is a maximum.

(a) Show that the point  $A(1, 0)$  lies on  $C$ .

(b) Find the coordinates of the point  $Q$ .

(c) Find the area of the shaded region between  $C$  and the line  $PQ$ .