1.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(a) Show that $\mathbf{AB} = c\mathbf{I}$, stating the value of the constant c.

(b) Hence, or otherwise, find $\mathbf{A}^{-1}$.

Q2.

. (a) Write down the 2×2 matrix which represents an enlargement with centre (0, 0) and scale factor k.

(1)

(b) Write down the 2×2 matrix which represents a rotation about (0, 0) through –90°.

(2)

(c) Find the 2×2 matrix which represents a rotation about (0, 0) through –90° followed by an enlargement with centre (0, 0) and scale factor 3.

(2)

The point A has coordinates $(a + 2, b)$ and the point B has coordinates $(5a + 2, 2 - b)$. A is transformed onto B by a rotation about (0, 0) through –90° followed by an enlargement with centre (0, 0) and scale factor 3.

(d) Find the values of a and b.

(5)

(Total 10 marks)

3

$$f(x) = 2x^3 - 8x^2 + 7x - 3.$$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

4 Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(*a*) write down the other root of the equation,

(1)

(*b*) find the value of *p* and the value of *q*.

(3)

5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3 .

Given that $z_1 = 3 + \text{i}$, find z_2 and z_3 .

(4)

6 (a) Show that $\sum_{r=1}^{n} (r^2 - r - 1) = \frac{1}{3}(n-2)n(n+2)$.

(6)

(b) Hence calculate the value of $\sum_{r=10}^{40} (r^2 - r - 1)$.

(3)
(Total 9 marks)

7. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(b) show that

$$\sum_{r=1}^{n} (r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where *a* and *b* are integers to be found.

(5)

(c) Hence show that

$$\sum_{r=n+1}^{2n}(r+2)(r+3)=\frac{1}{3}n\left(7n^2+27n+26\right)$$

(3)
(Total 14 marks)

8 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

(6)

9. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geq 1, \quad u_1 = 1.$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that $u_n = 2^n - 1$.

(5)

10 Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$

(6)

(b) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

11 A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$, is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2.$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}(4^n - 1).$$ **(5)**

12 $$f(n) = 2^n + 6^n.$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$. **(3)**

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

13 A sequence of numbers is defined by

$$u_1 = 2,$$

$$u_{n+1} = 5u_n - 4, \qquad n \geq 1.$$

Prove by induction that, for $n \in \mathbb{Z}$, $u_n = 5^{n-1} + 1$. **(4)**

14. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $\mathrm{f}(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$.

(7)

15. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}.$$