The Palmer Catholic Academy

Edexcel GCE **Pure 1 homework**

Time: 2 hours

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. Simplify

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1},$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

2. (*a*) Find the value of $8^{\frac{5}{3}}$.

(1)

(*b*) Simplify fully $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2}$. (2)

3.

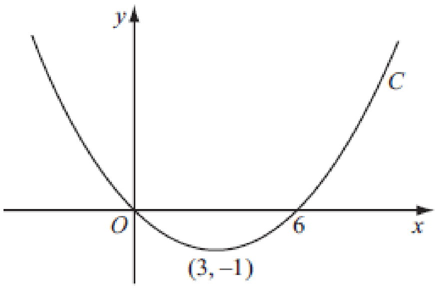

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through (6, 0).
The curve C has a minimum at the point (3, –1).

On separate diagrams, sketch the curve with equation

(*a*) $y = f(2x)$, **(3)**
(*b*) $y = -f(x)$, **(3)**
(*c*) $y = f(x + p)$, where p is a constant and $0 < p < 3$. **(4)**

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points. **(3)**

4. The curve C has equation $y = x^2(x - 6) + \dfrac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(*a*) Show that the length of PQ is $\sqrt{170}$. **(4)**
(*b*) Show that the tangents to C at P and Q are parallel. **(5)**
(*c*) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

5. (*a*) On the axes below sketch the graphs of

(i) $y = x(4 - x)$,

(ii) $y = x^2(7 - x)$,

showing clearly the coordinates of the points where the curves cross the coordinate axes. **(5)**

(*b*) Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$.

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(*c*) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)

6. Given that $\log_3 x = a$, find in terms of a,

(*a*) $\log_3 (9x)$

(2)

(*b*) $\log_3 \left(\frac{x^5}{81}\right)$

(3)

giving each answer in its simplest form.

(*c*) Solve, for x,

$$\log_3 (9x) + \log_3 \left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

(4)

7. (i) Solve, for $-180° \leq x < 180°$,

$$\tan(x - 40°) = 1.5,$$

giving your answers to 1 decimal place.

(3)

(ii) (*a*) Show that the equation

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1 = 0.$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\sin\theta \tan\theta = 3\cos\theta + 2,$$

showing each stage of your working.

(5)

8.

$$f(x) = 2x^3 + 3x^2 - 6x + 1.$$

(a) Find the remainder when $f(x)$ is divided by $(2x - 1)$. **(2)**

(b) *(i)* Find the remainder when $f(x)$ is divided by $(x + 2)$.

(ii) Hence, or otherwise, solve the equation

$$2x^3 + 3x^2 - 6x - 8 = 0,$$

giving your answers to 2 decimal places where appropriate. **(7)**

9.

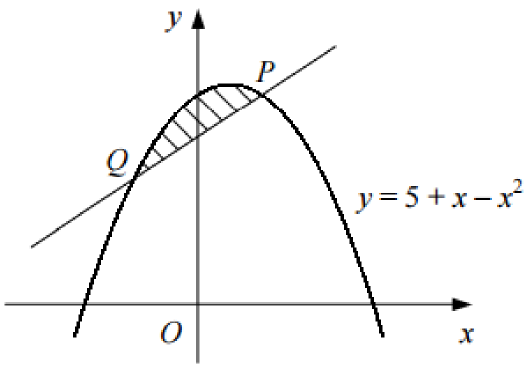

Figure 2

Figure 2 shows the curve with equation $y = 5 + x - x^2$ and the normal to the curve at the point P (1, 5).

(a) Find an equation for the normal to the curve at P in the form $y = mx + c$. **(5)**

(b) Find the coordinates of the point Q, where the normal to the curve at P intersects the curve again. **(2)**

(c) Show that the area of the shaded region bounded by the curve and the straight line PQ is $\frac{4}{3}$. **(6)**

10.

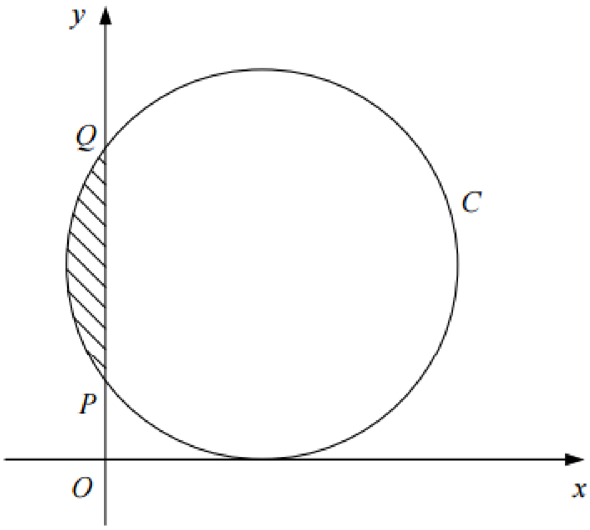

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 8x - 10y + 16 = 0.$$

(a) Find the coordinates of the centre and the radius of C. **(3)**

C crosses the y-axis at the points P and Q.

(b) Find the coordinates of P and Q. **(3)**

The chord PQ subtends an angle of θ at the centre of C.

(c) Using the cosine rule, show that $\cos \theta = \frac{7}{25}$. **(4)**

(d) Find the area of the shaded minor segment bounded by C and the chord PQ. **(4)**